Hollow cylinder :-

It is interesting to note that a hollow shaft has greater tortional rigidity than a solid shaft of the same material, mass and length.

Let us suppose that the couple on a hollow cylinder of length l, external and internal raddi r_2 and r_1 respectively is given by

$$\tau = \frac{2\pi\eta\theta}{l}\int_{r_1}^{r_2} x^3 \cdot dx$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{x^4}{4}\right]_{r_1}^{r_2}$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{r_2^4 - r_1^4}{4}\right]$$

$$\therefore C' \text{ (torsional rigidity) } = \frac{\eta\pi(r^{1/2} - r^{1/2})}{2l}$$

Similarly

C (torsional rigidity of a solid cylinder)

$$= \frac{\eta\pi r^4}{2l}$$

$$\therefore \frac{C'}{C} = \frac{r^{1/2} - r^{1/2}}{r^4} = \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4}$$

Since masses of the cylinder are equal.

$$\pi(r_2^2 - r_1^2)l\rho = \pi r^2 l\rho$$

$$\text{or, } r_2^2 - r_1^2 = r^2$$

$$\therefore \frac{C'}{C} = \frac{r_2^2 + r_1^2}{r^2}$$

$$= \frac{r_2^2 - r_1^2 + 2r_1^2}{r^2}$$

Putting $r_1^2 = -r_1^2 + 2r_1^2$

$$\therefore \frac{C'}{C} = \frac{r_1^2 + 2r_1^2}{r^2} = 1 + \frac{2r_1^2}{r^2}$$

will greater than one.

i.e. $\frac{c'}{c} > 1$

$\therefore c' > c$